LECTURE No. 35

Kinematic Indeterminacy of Structures

A structure is said to be kinematically indeterminate if the displacement components of its joints cannot be determined by compatibility conditions alone. In order to evaluate displacement components at the joints of these structures, it is necessary to consider the equations of static equilibrium. i.e. no. of unknown joint displacements over and above the compatibility conditions will give the degree of kinematic indeterminacy.

Fixed beam : Kinematically determinate :

Simply supported beam Kinematically indeterminate

Any joint – Moves in three directions in a plane structure
Two displacements $\delta x$ in $x$ direction, $\delta y$ in $y$ direction, $\theta$ rotation about $z$ axis as shown.

Roller Support :
$r = 1, \delta y = 0, \theta & \delta x$ exist – DOF = 2  $e = 1$

Hinged Support :
$r = 2, \delta x = 0, \delta y = 0, \theta$ exists – DOF = 1  $e = 2$

Fixed Support :
$r = 3, \delta x = 0, \delta y = 0, \theta = 0$  DOF = 0  $e = 3$

i.e. reaction components prevent the displacements ;:: no. of restraints = no. of reaction components.

Degree of kinematic indeterminacy :
Pin jointed structure :Every joint – two displacements components and no rotation
\[ \therefore \ Dk = 2j - e \quad \text{where,} \quad e = \text{no. of equations of compatibility} = \text{no. of reaction components} \]

**Rigid Jointed Structure:** Every joint will have three displacement components, two displacements and one rotation. Since, axial force is neglected in case of rigid jointed structures, it is assumed that the members are inextensible & the conditions due to inextensibility of members will add to the numbers of restraints. i.e to the ‘e’ value.

\[ \therefore \ Dk = 3j - e \quad \text{where,} \quad e = \text{no. of equations of compatibility} = \text{no. of reaction components} + \text{constraints due to in extensibility} \]

**Example 1:** Find the static and kinematic indeterminacies

\[ r = 4, \ m = 2, \ j = 3 \]

\[ Ds = (3m + r) - 3j = (3 \times 2 + 4) - 3 \times 3 = 1 \]

\[ Dk = 3j - e = 3 \times 3 - 6 = 3 \]

i.e. rotations at A, B, & C i.e. \( \theta_a, \theta_b \) & \( \theta_c \) are the displacements.

\( e = \text{reaction components} + \text{inextensibility conditions} = 4 + 2 = 6 \)

**Example 2:**

\[ Ds = (3m + r) - 3j = (3 \times 3 + 6) - 3 \times 4 = 3 \]

\[ Dk = 3j - e = \text{no. of reaction components} + \text{conditions of inextensibility} = 6 + 3 = 9 \]

\[ Dk = 3 \times 4 - 9 = 3 \quad \text{i.e. rotation} \ \theta_b, \theta_c \ & \text{sway.} \]
Example 3:

\[ D_s = (3m + r) - 3j \]

\[ r = 6, \ m = 10, \ j = 9 \]

\[ D_s = (3 \times 10 + 6) - 3 \times 9 = 9 \]

Conditions of inextensibility:
Joint: B C E F H I
\[ \begin{array}{cccccc}
1 & 1 & 2 & 2 & 2 & \text{Total} = 10
\end{array} \]

Reaction components \( r = 6 \)

\[ \therefore \ e = 10 + 6 = 16 \]

\[ \therefore \ D_k = 3j - e \]
\[ = 3 \times 9 - 16 = 11 \]