Lecture-49   

**FM Threshold Effect:**

The output signal to noise ratio of FM receiver is valid only if the carrier to noise ratio is measured at the discriminator input is high compared to unity. It is observed that as the input noise is increased and the carrier to noise ratio decreased, the FM receiver breaks. At first individual clicks are heard in the receiver output and as the carrier to noise ratio decreases still further, the clicks rapidly merge in to a crackling or sputtering sound.

Near the break point eqn8.50 begins to fail predicting values of output SNR larger than the actual ones. This phenomenon is known as the threshold effect.

The threshold effect is defined as the minimum carrier to noise ratio that gives the output SNR not less than the value predicted by the usual signal to noise formula assuming a small noise power.

For a qualitative discussion of the FM threshold effect, consider, when there is no signal present, so that the carrier is unmodulated. Then the composite signal at the frequency discriminator input is

\[
x(t) = [A_c + n_i(t)] \cos 2\pi f_c t - n_o(t) \sin 2\pi f_c t \tag{8.55}
\]

Where \(n_i(t)\) and \(n_o(t)\) are inphase and quadrature component of the narrow band noise \(n(t)\) with respect to carrier wave \(A_c \cos 2\pi f_c t\). The phasor diagram of fig8.17 below shows the phase relations b/n the various components of \(x(t)\) in eqn(8.55).

![Fig 8.17 A phasor diagram interpretation of eqn 8.55](image)

As the amplitudes and phases of \(n_i(t)\) and \(n_o(t)\) change randomly with time the point \(P\) wanders around the point \(Q\).

When the carrier to noise ratio is large \(n_i(t)\) and \(n_o(t)\) are small compared to \(A_c\), so that point \(P\) always around \(Q\). Thus the angle \(\Theta(t)\) small and within a multiple of \(2\pi\) radians.
The output signal to noise ratio of FM receiver is valid only if the carrier to noise ratio is measured at the discriminator input is high compared to unity. It is observed that as the input noise is increased so that the carrier to noise ratio decreased, the FM receiver breaks. At first individual clicks are heard in the receiver output and as the carrier to noise ratio decreases still further, the clicks rapidly merge in to a crackling or sputtering sound.

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Consider, when there is no signal present, so that the carrier is unmodulated. Then the composite signal at the frequency discriminator input is

\[ x(t) = [A_c + n_i(t)] \cos 2\pi f_c t - n_q(t) \sin 2\pi f_c t \]  

Where \( n_i(t) \) and \( n_q(t) \) are inphase and quadrature component of the narrow band noise \( n(t) \) with respect to carrier wave \( A_c \cos 2\pi f_c t \). The phasor diagram of fig8.17 below shows the phase relations b/n the various components of \( x(t) \) in eqn(8.55).

\[ x(t) \]
\[ \Theta(t) \]
\[ A_c \]
\[ n_i(t) \]
\[ n_q(t) \]
\[ \Psi(t) \]
\[ Q \]
\[ P \]

Fig 8.17 A phasor diagram interpretation of eqn 8.55

As the amplitudes and phases of \( n_i(t) \) and \( n_q(t) \) change randomly with time the point P wanders around the point Q.

When the carrier to noise ratio is large \( n_i(t) \) and \( n_q(t) \) are small compared to \( A_c \), so that point P always around Q. Thus the angle \( \Theta(t) \) small and within a multiple of \( 2\pi \) radians.
The point P occasionally sweeps around the origin and \( \Theta(t) \) increases or decreases by \( 2\pi \) radians. When the carrier to noise ratio is small. The clicks are produced only when \( \Theta(t) \) changes by \( \pm 2\pi \) radians.

From the phasor diagram of fig above, we may deduce the condition required for clicks to occur.

A positive going click occurs when the envelope \( r(t) \) and phase \( \Psi(t) \) of the narrow band noise \( n(t) \) satisfy the following conditions:

\[
\begin{align*}
  r(t) &> A_c \\
  \Psi(t) &< \pi < \Psi(t) + d\Psi(t) \\
  d\Psi(t)/dt &> 0
\end{align*}
\]

These conditions ensure that \( \Theta(t) \) changes by \( 2\pi \) radians in the time increment \( dt \), during which the phase of the narrow band noise increases by the incremental amount \( d\Psi(t) \).

Similarly, the condition for negative going click to occur are

\[
\begin{align*}
  r(t) &> A_c \\
  \Psi(t) &< -\pi < \Psi(t) + d\Psi(t) \\
  d\Psi(t)/dt &< 0
\end{align*}
\]

These conditions ensure that \( \Theta(t) \) changes by \( -2\pi \) radians in the time increment \( dt \).

As the carrier to noise ratio decreased, the average number of clicks per unit time increases. Whenthis number becomes large, the threshold is said to occur. Consequently, the output SNR deviates from a linearfunction of the carrier to noise ratio when the latter falls below the threshold.

This effect is shown in fig below, this calculation is based on the following two assumptions:

1. The output signal is taken as the receiver output measured in the absence of noise. The average output signal poweris calculated for a sinusoidal modulation that produces a frequency deviation \( \Delta f \) equal to \( 1/2 \) of the IF filter bandwidth \( B \). The carrier is thus enabled to swing back and forth across the entire IF band.

2. The average output noise power is calculated when there is no signal present, i.e., the carrier is unmodulated, with no restriction placed on the value of the carrier to noise ratio.
Fig 8.18 Variation of output SNR with carrier to noise ratio, demonstrating the FM threshold effect.

The curve plotted in fig above for the ratio \( (B/2W) = 5 \). The linear portion of the curve corresponds to the limiting value \( 3p(B/2W)^3 \). From the fig we may observe that the output SNR deviates appreciably from a linear function of the carrier to noise ratio \( p \), when \( p \) becomes less than a threshold of 10dB.

The threshold carrier to noise ratio \( p_m \) depends on the ratio of IF filter bandwidth to message bandwidth \( B/W \) and \( p_m \) is influenced by the presence of modulation.

We may state that the loss of message at an FM receiver output is negligible if the carrier to noise ratio satisfies the condition

\[
\frac{A^2_e}{2BN_0} \geq 10 \quad \text{------------------------8.56}
\]

\[
(SNR)_c \geq \frac{10B}{W} \quad \text{------------------------8.57}
\]

The IF filter bandwidth \( B \) is designed to equal the FM transmission bandwidth. Hence, we may use Carson's rule to relate \( B \) to the message bandwidth \( W \) as follows

\[
B = 2W(1+D)
\]

where \( D \) is the deviation ratio, for sinusoidal modulation, the modulation index \( \beta \) is used in place of \( D \). Therefore, for no significant loss of message at an FM receiver output as,
\[(\text{SNR})_c \geq 20(1+D)\]  \hspace{1cm} \text{---8.58}

or in terms of decibels,
\[10\log_{10}(\text{SNR})_c \geq 13 + 10 \log_{10}(1+D)\]  \hspace{1cm} \text{---8.59}

**FM threshold reduction:**

In specific applications such as space communications, it is required to reduce the noise threshold in an FM receiver so as to satisfactorily operate the receiver with the minimum signal power possible.

This can be achieved by using an FM demodulator with negative feedback (FMFB) or by using a phase locked loop demodulator.

![Diagram of FMFB demodulator](image)

**Fig 8.19 FMFB demodulator**

Fig above is a block diagram of an FMFB demodulator. The conventional local oscillator is replaced by VCO. To understand the operation of this receiver, suppose that VCO is removed from the circuit and the feedback path is left open.

Assume that the wide band FM is applied to the receiver input, and a second FM from the same source but with a modulation index a fraction smaller is applied to the VCO terminal of the product modulator. The output of the product modulator consists of sum and difference frequency components. The IF filter is designed to pass only difference frequency component. The frequency deviation of the IF filter output would be small, although the frequency deviation of both input FM wave is large, since the difference between their instantaneous deviations is small. Hence, the modulation indices would subtract, and the resulting FM wave at the IF filter output have a smaller modulation index than the input FM waves. This means that the IF filter bandwidth in fig above need only be a fraction of that required for either wideband FM wave. The FM wave with reduced modulation index passed by the IF filter is then frequency-demodulated by the combination of limiter/discriminator and finally...
processed by the base band filter. It is now apparent that second wide band FM waves replaced by VCO feed by o/p of low pass filter as in fig above.

Now, it will be shown that the SNR of an FMFB receiver is same as that of conventional FM receiver with the same input signal and noise power if the carrier to noise ratio is sufficiently large. Assume for the moment there is no feedback around the demodulator. In the combined presence of an unmodulated carrier $A_c \cos 2\pi f_c t$ and a narrow band noise

$$n(t) = n_i(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t,$$

the phase of the composite signal $x(t)$ at the limiter - discriminator input is approximately equal to $n_Q(t)/A_c$. This assumes that the carrier to noise ratio is high. The envelope of $x(t)$ is of no interest to us, because the limiter removes all variations in the envelope. Thus the composite signal at the frequency discriminator input consists of a small index phase - modulated wave with the modulation derived from the component $n_Q(t)$ of noise that is in phase quadrature with the carrier. When feedback is applied, the VCO generates a wave that reduces the phase - modulation index of the wave at the IF filter output, i.e., the quadrature component $n_Q(t)$ of noise.

Thus we see that as long as the carrier - to noise ratio is sufficiently large, the FMFB receiver does not respond to the in-phase noise component $n_i(t)$, but it would demodulate the quadrature noise like a signal. Signal and quadrature noise are reduced in the same proportion by the applied feedback, with the result that the base band SNR is independent of feedback. For large carrier to noise ratio the baseband SNR of an FMFB receiver is same as that of a conventional receiver.

The FMFB users a very important piece of a priori information that even though the carrier frequency of the incoming FM wave will usually have large frequency deviations, its rate of change will be at the base band rate.

An FMFB demodulator is essentially a tracking filter that can track only the slowly varying frequency of wideband FM waves. Consequently it responds only to a narrow band of noise centered about the instantaneous carrier frequency. The bandwidth of noise to which the FMFB receiver responds is precisely the band of noise that the VCO tracks. As a result, FMFB receivers allow a threshold extension. Like the FMFB demodulator, the PLL is also a tracking filter and hence it also provides threshold extension.