**Design of combined footing – Slab and Beam type-2**

Design a rectangular combined footing with a central beam for supporting two columns 400x400 mm in size to carry a load of 1000kN each. Center to center distance between the columns is 3.5m. The projection of the footing on either side of the column with respect to center is 1m. Safe bearing capacity of the soil can be taken as 190kN/m². Use M20 concrete and Fe-415 steel.

Solution: Data

- $f_{ck} = 20\text{N/mm}^2$
- $f_y = 415\text{mm}^2$
- $f_b (\text{SBC}) = 190\text{ kN/m}^2$
- Column A = 400 mm x 400 mm,
- Column B = 400 mm x 400 mm,
- c/c spacing of columns = 3.5,
- $P_A = 1000\text{ kN}$ and $P_B = 1000\text{ kN}$

**Required:** To design combined footing with central beam joining the two columns.

**Ultimate loads**

$P_{uA} = 1.5 \times 1000 = 1500\text{ kN}$, $P_{uB} = 1.5 \times 1000 = 1500\text{ kN}$

**Proportioning of base size**

- Working load carried by column A = $P_A = 1000\text{ kN}$
- Working load carried by column B = $P_B = 1000\text{ kN}$
- Self weight of footing 10\% x ($P_A + P_B) = 200\text{ kN}$
- Total working load = 2200\text{ kN}
- Required area of footing $A_f = \frac{\text{Total load}}{\text{SBC}} = \frac{2200}{190} = 11.57\text{ m}^2$
- Length of the footing $L_f = 3.5 + 1 + 1 = 5.5\text{ m}$
- Required width of footing $b = \frac{A_f}{L_f} = \frac{11.57}{5.5} = 2.1\text{ m}$
- Provide footing of size 5.5 x 2.1 m

For uniform pressure distribution the C.G. of the footing should coincide with the C.G. of column loads. As the footing and columns loads are symmetrical, this condition is satisfied.
The details are shown in Figure

Total load from columns = \( P = (1000 + 1000) = 2000 \) kN.
Upward intensity of soil pressure=Column loads, \( P/A_f = 2000/5.5 \times 2.1 = 173.16 \) kN/m²<br>

**Design of slab:**

Intensity of Upward pressure = \( p = 173.16 \) kN/m²
Consider one meter width of the slab (\( b=1\)m)
Load per m run of slab at ultimate = \( 173.16 \times 1 = 173.16 \) kN/m
Cantilever projection of the slab (For smaller column) = 1050 - 400/2 = 850mm
Maximum ultimate moment = \( 173.16 \times 0.850^2/2 = 62.55 \) kN-m. (Working condition)

For M20 and Fe 415, \( Q_u \max = 2.76 \) N/mm²
Required effective depth = \( \sqrt{(62.15 \times 1.5 \times 10^6/(2.76 \times 1000))} = 184.28 \) mm
Since the slab is in contact with the soil clear cover of 50 mm is assumed.
Using 20 mm diameter bars, effective cover = 20/2 + 50 say 75 mm
Required total depth = 184.28 + 75 = 259.4 mm. However provide 300 mm from shear consideration as well. Provided effective depth = \( d = 300-75 = 225 \) mm

**To find steel**

\( M_u/ bd^2 = 1.5 \times 62.15 \times 10^6/1000 \times 225^2 = 1.84 < 2.76, \) URS
\( P_t = 0.584 \% \)
A_{st} = 1314 \text{ mm}^2 \\
Use \#20 \text{ mm} \text{ diameter bar at spacing = } 1000 \times 1314 / 1314 = 238.96 \text{ mm} \text{ say } 230 \text{ mm c/c} \\
Area provided = 1000 \times 314 / 230 = 1365 \text{ mm}^2. \text{ Hence safe. This steel is required for the entire length of the footing.} \\

Check the depth from one-way shear considerations \\
Design shear force = V_u = 1.5 \times 173.16 x(0.850-0.225) = 162.33 \text{ kN} \\
Nominal shear stress = \tau_v = V_u / bd = 162330 / (1000 \times 225) = 0.72 \text{ MPa} \\
Permissible shear stress \ 
\Pi_t = 100 \times 1365 / (1000 \times 225) = 0.607 \% \text{, } \tau_{uc} = 0.51 \text{ N/mm}^2 \\
Value of k for 300 mm thick slab = 1 \\
Permissible shear stress = 1 \times 0.51 = 0.51 \text{ N/mm}^2 \\
\tau_{uc} < \tau_v \text{ and hence unsafe.} \\

The depth may be increased to 400 mm so that \text{d} = 325 \text{mm} \\

M_{ld} / bd^2 = 1.5 \times 62.15 \times 10^6 / 1000 \times 325^2 = 0.883 < 2.76, \text{ URS} \\
\Pi_t = 0.26 \% \text{, } A_{st} = 845 \text{ mm}^2 \\
Use \#16 \text{ mm diameter bar at spacing = } 1000 \times 201 / 845 = 237.8 \text{ mm}, \text{ say } 230 \text{ mm c/c} \\
Area provided = 1000 \times 201 / 230 = 874 \text{ mm}^2. \\

Check the depth from one-way shear considerations \\
Design shear force = V_u = 1.5 \times 173.16 x (0.850-0.325) = 136.36 \text{ kN} \\
Nominal shear stress = \tau_v = V_u / bd = 136360 / (1000 \times 325) = 0.42 \text{ MPa} \\
Permissible shear stress \ 
\Pi_t = 201 \times 1000 / 150 = 1340 \text{ mm}^2 \text{ and } \Pi_t = 0.41 \% \text{ and hence } \tau_{uc} = 0.45 \text{ MPa and safe.} \\

Check for development length \\
L_{df} = 47 \times \text{times diameter = 47} \times 16 = 768 \text{ mm} \\
Available length of bar = 850 - 25 = 825 \text{mm} > 768 \text{ mm and hence safe.} \\

Transverse reinforcement \\
Required A_{st} = 0.12 \times bD / 100 = 0.12 \times 1000 \times 400 / 100 = 480 \text{mm}^2 \\
Using 10 mm bars, spacing = 1000 \times 79 / 480 = 164.58 \text{mm} \\
Provide distribution steel of \#10 \text{ mm at } 160 \text{ mm c/c} \\

Design of Longitudinal Beam \\
Two columns are joined by means of a beam monolithic with the footing slab. The load from the slab will be transferred to the beam. As the width of the footing is 2.1 m, the net upward soil pressure per meter length of the beam under service. \\
\text{= w} = 173.16 \times 2.1 = 363.64 \text{ kN/m} \\

Shear Force and Bending Moment at service condition \\
V_{AC} = 363.64 \times 1 = 363.14 \text{ kN}, \text{ V}_{AB} = 1000 - 363.14 = 636.36 \text{ kN} \\
V_{BD} = 363.14 \text{ kN}, \text{ V}_{BA} = 636.36 \text{ kN}
Point of zero shear is at the center of footing at L/2, i.e., at E
Maximum B.M. occurs at E
\[ M_E = 363.64 \times \frac{2.75^2}{2} - 1000 (2.75 - 1) = -375 \text{ kN.m} \]

Bending moment under column A = \( M_A = 363.64 \times \frac{1^2}{2} = 181.82 \text{ kN.m} \)
Similarly bending moment under column B = \( M_B = 181.82 \text{ kN-m} \)
Let the point of contra flexure be at a distance \( x \) from C
Then, \( M_x = 363.63x^2/2 - 1000(x-1) = 0 \)
Therefore \( x = 1.30 \text{ m and 4.2m from C} \)

**Depth of beam from B.M. Considerations:**
The width of beam is kept equal to the maximum width of the column i.e. 400 mm. Determine the depth of the beam from absolute maximum BM. This is in the central part where T-beam action is available. Assume the beam as rectangular at the center of span where the moment is maximum, we have,

\[ d = \sqrt{\frac{375 \times 1.5 \times 10^6}{(2.76 \times 400)}} = 713.8 \text{ mm} \]
Provide total depth of 800 mm. Assuming two rows of bars at an effective cover of 75 mm, the effective depth provided = \( d = 800 - 75 = 725 \text{ mm} \).
Check the depth for Two-way Shear:

The column B can punch through the footing only if it shears against the depth of the beam along its two opposite edges, and along the depth of the slab on the remaining two edges. The critical section for two-way shear is taken at distance $d/2$ (i.e. 680/2 mm) from the face of the column. Therefore, the critical section will be taken at a distance half the effective depth of the slab ($d_s/2$) on the other side as shown in Fig.
In this case \( b = D = 400 \text{ mm}, \ d_b = 725 \text{ mm}, \ d_s = 325 \text{ mm} \)

Area resisting two-way shear
\[
= 2(b \times d_b + d_s \times d_s) + 2(D + d_b)ds
\]
\[
= 2(400 \times 725 + 325 \times 325) + 2(400 + 725) \times 325 = 1522500 \text{ mm}^2
\]

Design shear = \( P_{ud} = \text{column load} - W_u \times \text{area at critical section} \)
\[
= 1500 - 173.16 \times 1.5 \times (0.400 + 0.325) \times (0.400 + 0.725)
\]
\[
= 1500 - 173.16 \times 1.5 \times (0.400 + 0.325) \times (0.400 + 0.725)
\]
\[
= 1288.14 \text{ kN}
\]

Shear stress resisted by concrete = \( \tau_{uc} = \tau_{uc} \times K_s \)
where, \( \tau_{uc} = 0.25 \sqrt{f_{ck}} = 0.25 \sqrt{20} = 1.11 \text{ N/mm}^2 \)
\( K_s = 0.5 + d / D = 0.5 + 400/400 = 1.5 \neq 1 \) Hence \( K_s = 1 \)
\( \tau_{uc} = 1 \times 1.11 = 1.11 \text{ N/mm}^2 \), Therefore safe

**Area of Reinforcement**

Cantilever portion BD and AC
Length of cantilever from the face of column = 0.8 m.
Ultimate moment at the face of column = \( 363.64 \times 1.5 \times 0.8^2 / 2 = 177.53 \text{ kN-m} \)
\[
M_{um} = 2.76 \times 400 \times 725^2 \times 10^{-6} = 580.29 \text{ kN.m} > 177.53 \text{ kN-m}
\]
Therefore Section is singly reinforced.

\[ M_0/bd^2 = 177.53 \times 10^6 / (400 \times 725^2) = 0.844 < 2.76, \text{ URS} \]
P=0.248\%, A_{st} =719.2 \text{ mm}^2

Provide 4 - 16 mm at bottom face, Area provided = 804 \text{ mm}^2, p_t=0.278\%
L_d = 47x16 = 752 \text{ mm}

Curtailment: All bottom bars will be continued up to the end of cantilever for both columns. If required two bottom bars of 2-16mm will be curtailed at a distance d (= 725 mm) from the point of contra flexure in the portion BE as shown in figure.

**Region AB between points of contra flexures**

The beam acts as an isolated T- beam.

\[ b_t = \left[ \frac{L_o}{(L_o/b + 4)} \right] + b_w \text{ where,} \]
\[ L_o = 4.2-1.3=2.9 \text{ m} = 2900 \text{ mm} \]
\[ b = \text{actual width of flange} = 2100 \text{ mm}, b_w = 400 \text{ mm} \]
\[ b_t = \frac{2900}{(2900 / 2100) + 4} + 400 =938.9 \text{mm} \]
\[ D_t = 400 \text{ mm}, \quad M_u = 1.5 \times 375=562.5 \text{ kN-m} \]

Moment of resistance \( M_{af} \) of a beam for \( x = D \) is:
\[ (M_{af}) = [0.36 \times 20 \times 938.9 \times 400 (725 - 0.42\times400)] \times 10^6 \]
\[ = 1506 \text{ kN.m} > M_u \quad (= 562.5 \text{ kN-m}) \]

Therefore \( X_u < D_t \)
\[ M_u=0.87f_yA_{st}(d-f_yA_{st}/f_{ck}b_t) \]
\[ A_{st} = 2334 \text{ mm}^2 \]

Provide 4 bars of 25 mm and 2 bars of 16 mm,
Area provided = 2354 \text{ mm}^2 \> 2334 \text{ mm}^2
\[ p_t = 100 \times 2334/(400x725) = 0.805 \% \]

Curtailment: Curtailment can be done as explained in the previous problem. However extend all bars up to a distance ‘d’ from the point of contra flexure i.e up to 225 mm from the outer faces of the columns. Extend 2-16mm only up to the end of the footing.

**Design of shear reinforcement**

**Portion between column i.e. AB**

In this case the crack due to diagonal tension will occur at the point of contra flexure because the distance of the point of contra flexure from the column is less than the effective depth \( d(= 725 \text{mm}) \)

(i) Maximum shear force at A or B = \( V_{umax} = 1.5 \times 636.36 = 954.54 \text{ kN} \)

Shear at the point of contra flexure = 954.54-1.5x 363.64x0.3 = 790.9 kN
\[ \tau_c = \frac{790900}{(400x725)} = 2.73 \text{ MPa} < \tau_{c,max}(2.8 \text{ MPa}) \]

Area of steel available = 2354 mm\(^2\), 0.805 \%
\[ \tau_c = 0.59 \text{MPa}, \quad \tau_v > \tau_c \]

Design shear reinforcement is required.

Using 12 mm diameter 4 - legged stirrups,
Spacing = 0.87 \times 415 \times (4 \times 113) /(2.73-0.59)x400 = 190.6 mm say 190 mm c/c

Zone of shear reinforcements is between \( \tau_v \) to \( \tau_c \) = \( m \) from support B towards \( A \)

**Cantilever portion BD and AC**

\[ V_{umax} = 363.64 \times 1.5 = 545.45 \text{ kN}, \]
Shear from face at distance \( d = V_{ud} = 545.45-363.64 \times 1.5(0.400 / 2 + 0.725) = 40.90 \text{ kN} \)
\[ \tau_v = \frac{40900}{(400x725)} = 0.14 \text{ MPa} < \tau_{c,max}. \text{ This is very small} \]
Steel at this section is 4 – 16 mm, Area provided = 804 mm$^2$, $\rho = 0.278\%$
$$\tau_c = 0.38 \text{N/mm}^2$$ (Table IS:456-2000). No shear steel is needed.
Provide minimum steel.
Using 12 mm diameter 2-leg stirrups,
Spacing = $0.87 \times 415 \times (2 \times 113) / (0.4 \times 400) = 509.9$ mm say 300 mm c/c

![Diagram showing reinforcement layout with dimensions and annotations]